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Estimate Mathematical Model to Calculate the View Angle Depending on the Camera Zoom

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Abstract— A mathematical model to estimate the camera view angle for a certain object has been found based on camera zoom where the fitting curves for the practical data of the view angles ($\theta_h, and\ \theta_w$) in the image plane which decreased with increasing distance (D) for each zoom number(Z) of the used camera were achieved. Then find the mathematical modeling equation that relates view angles ($\theta_h, and\ \theta_w$), real distance (l_r) and zoom number (Z) the comparison between theoretical estimation and practical result for the camera view angles and zoom number give a very good agreement between them where the estimated vertical and horizontal camera view angles very close to the real measurements.

Index Terms— View angle, camera resolution, mathematical model, estimated view angle.

1 Introduction

Throughout the centuries, people have tried to find a quicker and better way to capture the world around them. Drawing and painting were the most visible forms of this type of endeavor. Since not everyone is a skilled craftsman or artist when it comes to faithful renditions, instruments have been invented to make the rendering process easier. One of the earliest of these instruments is the camera obscure which literally means dark room built by Ibn Al-Haytham the `father of optics'. This tool started as a full-sized darkened room with a small opening on the end; it was used to observe solar eclipses and to aid artists in understanding perspective. Therefore, the first image capture devices required the observer to record the images by hand. Eventually, images projected through a small opening were miniaturized and improved through the use of lenses. These lenses made the image sharper and were able to resolve more details. Later, mirrors were added to a portable camera obscure, which facilitated the tracing of natural subjects. This invention became known as the camera Lucida [1]. Photography is the production of visible images, on a specially prepared surface, using light or radiation. The word comes from two Greek words that mean "writing with light". In other word it's the art, science and practice of creating images by recording light or other electromagnetic radiation, either electronically by means of an image sensor or chemically by means of a light-sensitive material such as photographic film [2].

The cameras map surrounding space through optical systems (lenses, mirrors, filters, etc.) on to photo-sensitive devices (e.g., film, CCD, CMOS sensors). Various combinations of optical elements lead to various types of cameras, to standard or non-standard ones. Sometimes it is an exigency raised from real world and sometimes just a challenge of intellectual curiosity to understand the geometry of various types of cameras [3, 4].

Many Researchers have studied the possibility of determining view angle camera and used the techniques of variety physical foundations. Here are reviews for some of these studies.

Steven Maxwell Seitz, (1997), study addresses the problem of synthesizing images of real scenes under three-dimensional transformations in viewpoint and appearance. Solving this problem enables interactive viewing of remote scenes on a computer [5].

Ming-Chih L. et al. (2010) presented an image-based system for measuring objects on an oblique plane based on pixel variation of CCD images for digital cameras by referencing to two arbitrarily designated points in image frames, based on an established relationship between the displacement of the camera movement along the photographing direction and the difference in pixel counts between reference points in the images [6].

Kouskouridas R. et al. (2012) proposed a novel algorithm for objects' depth estimation; moreover, they comparatively study two common two-part approaches, namely the Scale Invariant Feature Transform SIFT and the speeded-up robust features algorithm, in the particular application of location assignment of an object in a scene relatively to the camera, based on the proposed algorithm. [7].

2 Camera Resolution

Camera resolution was described the ability of an imaging system to resolve detail of the object that is being imaged. An imaging system may have many individual components including a lens and recording and display components. Each of

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these contributes to the optical resolution of the system, as will the environment in which the imaging is done [8]. Digital zoom (DZ) is a method of decreasing (narrowing) the apparent angle of view of a digital photographic or video image. The (DZ) is accomplished by cropping an image down to a centered area with the same aspect ratio as the original, and usually also interpolating the result back up to the pixel dimensions of the original. It is accomplished electronically, with no adjustment of the camera's optics, and no optical resolution is gained in the process [9]. When comparing the image quality achieved by digital zoom with image quality achieved by resizing the image in post-processing, there's a difference between cameras that perform potentially lossy image compression like JPEG, and those that save images in an always lossless Raw image format. In the former case, digital zoom (DZ) tends to be superior to enlargement in post-processing, because the camera may apply its interpolation before detail is lost to compression. In the latter case, resizing in post-production was yields results equal or superior to digital zoom. Some digital cameras rely entirely on digital zoom, lacking a real zoom lens, as on most camera phones. Other cameras do have a real zoom lens, but apply digital zoom automatically once its longest focal length has been reached. Professional cameras generally do not feature digital zoom [9].

3 Compute Camera View Angles

A digital cannon camera (power shot A800 and IXUS digital compact camera, 2011), as shown in Fig. (1), with technical specifications tabulated in table 1, has been used in this study [10].

Fig. 1. Cannon camera (power shot A800 and IXUS digital compact camera).

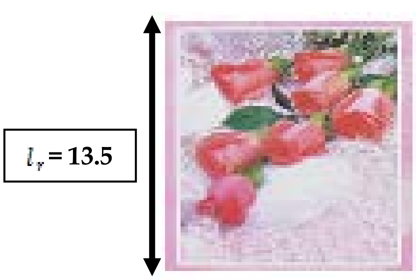

Fig. 2. The image of object (picture).

TABLE 1

Technical specification of cannon camera [10]

Optical zoom	3.3x optical zoom, 6.6–21.6mm (35mm equivalent: 37–122 mm), f/3.0-5.8
Resolution	10.0Mp
Sensor type , size	CCD , 1/2.3in
Max image size	3648 x 2736
Focusing system ,points ,distance	TTL, AiAF (Face Detection / 5-point), 1-point AF (fixed centre), 1cm (W) from front of lens in macro
ISO sensitivity	ISO100 – 1600
Monitor	2.5in TFT, approx. 115,000 dots

First the object (picture) shown in Fig. (2), is placed in front of the camera then the geometry of the scene sets object plane parallel to the image plane. For each zoom number (Z) of the camera (7, 8, 10, 12, 15 and 22) have been captured image for the object (with 10 Megapixels size) at different distances (D= 1 m to10 m) with changing 1m in each step. Then built an algorithm using MATLAB language to calculate the object's length in pixels (picture's height) for each captured image and this is performed by determine the length between two ends of the object (picture) manually using computer mouse.
According to the following relationship: can be find image scale factor plane (Scf) [11].

$$Scf = \frac{l_p}{l_r} \ldots\ldots\ldots(1)$$

Where l_p: object's length in image pixel plane; and the l_r: real object length in (cm).
By using eq. (1) can be computing vertical and horizontal scene viewing in the capture image in (cm) based on the following sq.'s:

$$H = Scf * Ih \ldots\ldots(2)$$

$$W = Scf * Iw \ldots\ldots(3)$$

Where Ih and Iw are the image length and width respectively in pixels. Then calculating the camera view angles in terms of H and W at each distance D and for each zoom number Z, according to the following eq.'s:

$$\theta_h = 2 * tan^{-1}\left(\frac{H}{2D}\right) \ldots\ldots\ldots\ldots(4)$$

$$\theta_w = 2 * tan^{-1}\left(\frac{W}{2D}\right) \ldots\ldots\ldots(5)$$

Where D in a eq.'s (4) and (5) represents the distance between the cameras and the object θ_h and θ_w : represent vertical and horizontal camera view angles respectively, as shown in Fig.(3).

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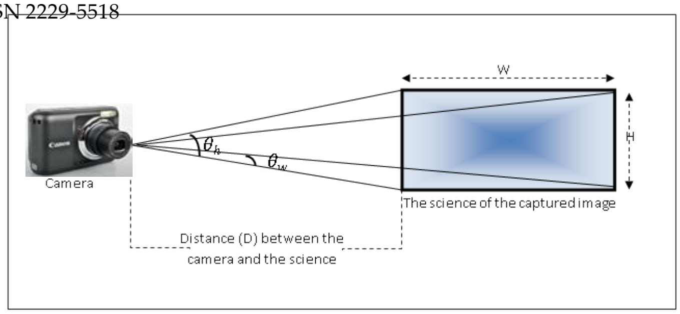

FIG. 2. VERTICAL AND HORIZONTAL CAMERA VIEW ANGLES (θ_h AND θ_w)

($\theta_h, and\ \theta_w$) for each captured images. Then draw the relation between zoom number (Z) and the view angles ($\theta_h, and\ \theta_w$) for different distances (D) see Fig.(4) and Fig.(5) . From these two figures can be shown that the relationship the camera zoom (Z) and the vertical and horizontal view angle ($\theta_h, and\ \theta_w$). Where can be see that the viewing angles ($\theta_h, and\ \theta_w$) are decreased in non-linear relation with increased zoom number (z) for all distance between the camera and the object (D).

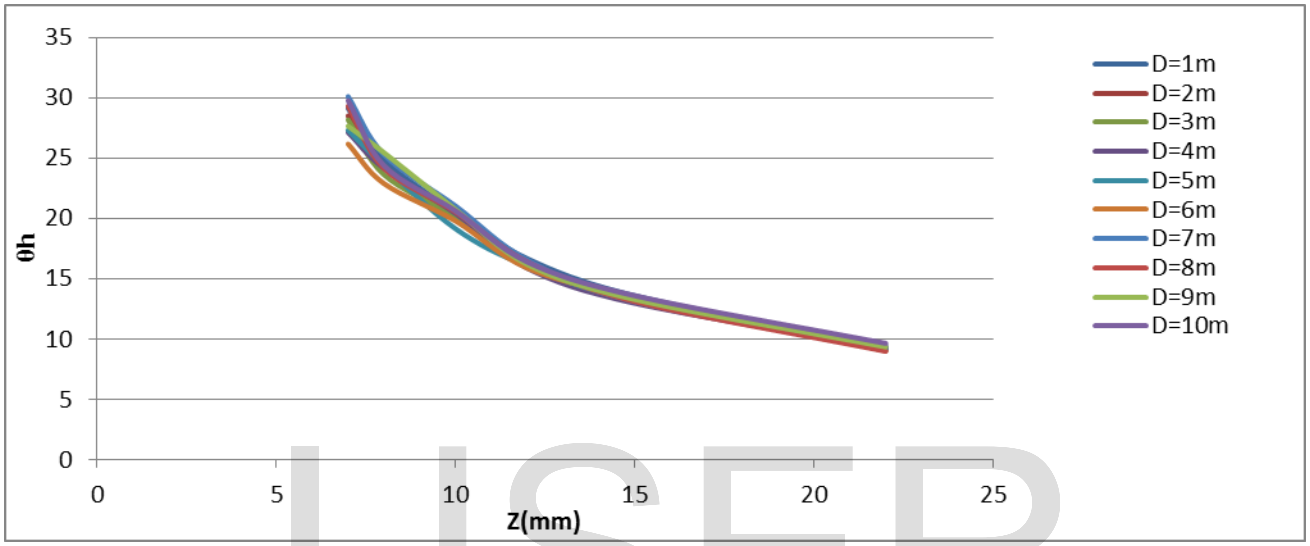

Fig.4: The relation between zoom number (Z mm) and vertical view angle (θ_h)

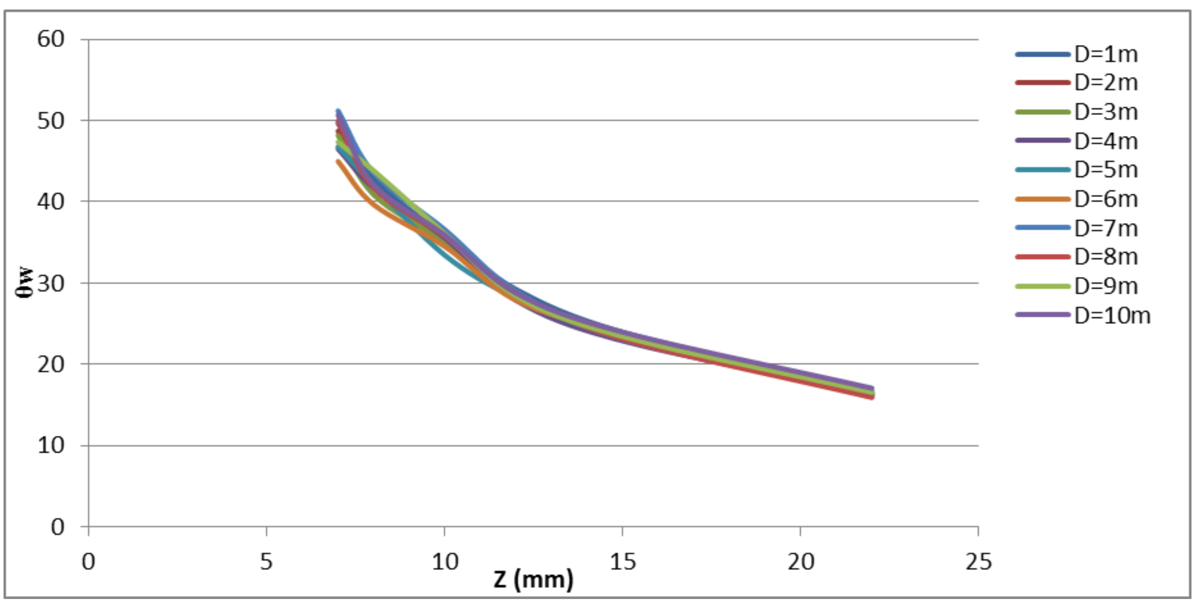

Fig. 5: The relation between zoom number (Z mm) and horizontal view angle (θ_w)

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A graph between zoom number (Z) and the ratio between (θ_h) and (θ_w) (eq.6) was plotted for each distance (D) in Fig. (6).

$$R = \frac{\theta_h}{\theta_w} \ldots \ldots . (6)$$

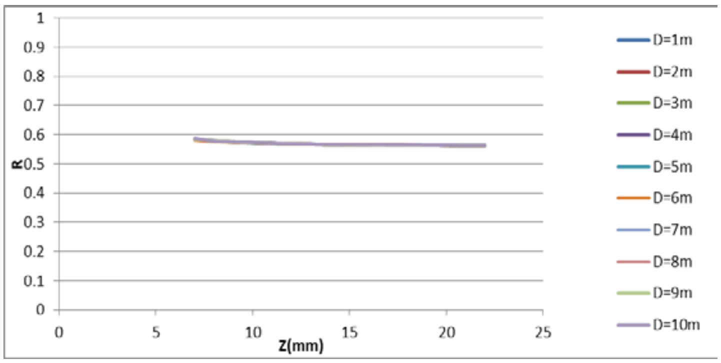


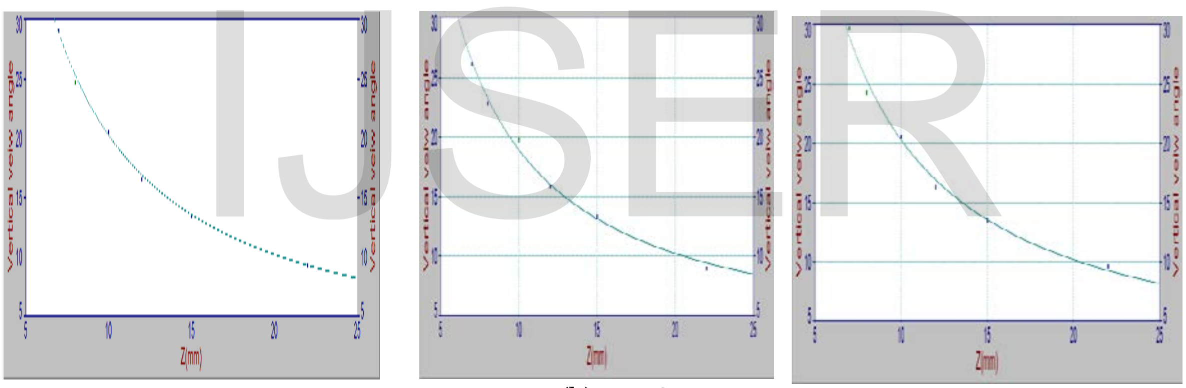
Fig.6: The relation between (Z) and (R)

This graph shows the relationship between the (Z) and the ratio (R) between vertical and horizontal view angles for different distances (D). Can be observed that this ratio nearly constant for all the different camera zoom (Z) and distances (D). Where the ratio value approximately (0.58).

By using Table Curve "2D version 5.01" software to estimate the best fitting equations between the viewing angle (θ_i) and the zoom number (Z), get the best fitting equation as follow:

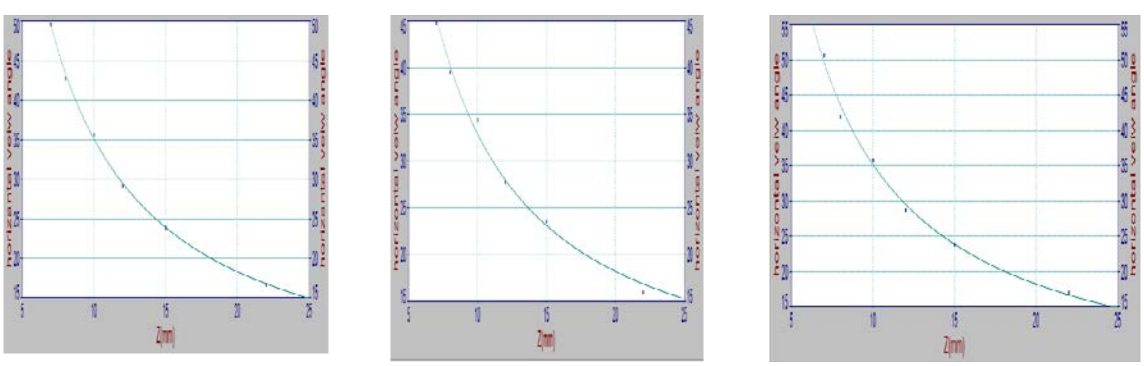
$$\theta_i = a_i + \frac{b_i}{Z} \ldots \ldots . (7)$$

Where θ_i represent either θ_h or θ_w, the estimated fitting equation curves for some cases (D=1, 6, and 10) θ_h shown in Fig. (7), and for θ_w shown in Fig. (8) for different distance (D). Of the equation (7) can be note the Inverse relationship between the view angle and camera zoom.



(a): D=1 m (b): D=6 m (c): D=10 m

Fig.7: Fitting curve of vertical view angle (θ_h) with zoom (Z) at some different distance (a,b,c): D= 1,6,10 m respectively.



(a): D=1 (b): D=6 (c): D=10

Fig.8: Fitting curve of horizontal view angle (θ_w) with zoom (Z) at some different distance (a,b,c): D= 1,6,10 m respectively.

TABLE 2

THE VALUES OF VERTICAL VIEW ANGLE (θ_h) WITH (Z) FOR EACH DISTANCE (D)

Zoom (Z mm) / Distance (Dm)	7	8	10	12	15	22
1	29.13	24.83	20.51	16.66	13.59	9.43
2	28.52	24.38	20.00	16.39	13.23	9.28
3	28.19	23.64	20.10	15.99	13.14	9.15
4	27.15	24.16	20.41	15.87	12.97	9.23
5	27.32	24.38	19.14	16.06	13.14	9.09
6	26.18	22.90	19.78	15.88	13.41	9.02
7	30.12	25.30	21.02	16.55	13.10	9.25
8	29.32	24.16	20.53	16.39	13.10	8.98
9	27.66	25.46	20.72	16.19	13.27	9.34
10	29.78	24.38	20.62	16.39	13.59	9.64

TABLE 3

THE VALUES OF WIDTH VIEW ANGLE (θ_w) WITH (Z) FOR EACH DISTANCE (D)

Zoom (Z mm) / Distance (Dm)	7	8	10	12	15	22
1	49.67	42.82	35.73	29.24	23.97	16.72
2	48.71	42.09	34.87	28.77	23.34	16.45
3	48.20	40.88	35.04	28.10	23.19	16.22
4	46.54	41.73	35.55	27.88	22.90	16.37
5	46.81	42.09	33.43	28.21	23.19	16.11
6	44.99	39.67	34.51	27.91	23.65	16.00
7	51.21	43.58	36.58	29.05	23.12	16.41
8	49.96	41.73	35.75	28.77	23.12	15.93
9	47.36	43.84	36.08	28.43	23.42	16.56
10	50.68	42.09	35.90	28.77	23.97	17.08

The two parameters a_i, and b_i, the notation (*i* either *h* or *w*) are tabulated in table (4) for (θ_h), and in table (5) for (θ_w) for all distances (D).

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TABLE 4

THE PARAMETERS OF THE fiTTING EQUATION OF VERTICAL VIEW ANGLE (θ_h) AS A FUNCTION OF (Z) FOR EACH DISTANCE (D)

Distance (D) in m	a_h-parameter	b_h-parameter	r^2-parameter
1	0.22	200.28	0.998
2	0.21	196.16	0.999
3	0.32	192.29	0.995
4	0.69	187.58	0.994
5	0.47	188.76	0.999
6	1.57	173.56	0.994
7	-0.85	213.76	0.996
8	-0.43	204.25	0.994
9	0.45	195.23	0.993
10	0.11	201.94	0.992
	Average =0.27	Average =195.38	

Table 5

The parameters of the fitting equation of horizontal view angle (θw) as a function of zoom (Z) for each distance (D)

Distance (D) in m	aw-parameter	bw-parameter	r2-parameter
1	1.55	335.22	0.999
2	1.47	329.12	0.999
3	1.60	323.15	0.996
4	2.18	315.84	0.993
5	1.81	317.69	0.999
6	3.60	293.19	0.993
7	-0.20	357.05	0.997
8	0.41	342.28	0.995
9	1.84	327.82	0.992
10	1.40	337.47	0.994
	Average =1.57	Average =327.88	

We took the average and matching between the actual data in table (2) and (3) theory using the equation derived after compensation rate values represent either i saluting the ai, bi where i (h or w) After the compensation rate values ai, bi and comparing the resulting values with practical values.

For checking the estimated models have been canceled the practical results for distances D=2m and 3m from fitting process in order to determine the values theoretically from the estimated mathematical models eq.'s (5,6) and make a comparison between the practical and theoretical values. There is an excellent agreement between them, as shown in tables (4, 5).

TABLE 6

THE RESULTS OF THE ESTIMATED ΘH AS COMPARED TO THE PRACTICAL VALUES AT SAME DISTANCE (D=2M AND 3M).

Distance D (m)	Z (m m)	Practical view angle (θ_h)	Estimate view angle (θ_h)	Absolute percentage error
2	7	28.52	28.19	0.0116
	8	24.38	24.70	0.0131
	10	20.00	19.81	0.0093
	12	16.39	16.56	0.010
	15	13.23	13.30	0.0053
	22	09.28	09.16	0.0130
3	7	28.19	28.19	0.0001
	8	23.64	24.70	0.0448
	10	20.10	19.81	0.0143
	12	15.99	16.56	0.0351
	15	13.14	13.30	0.0119
	22	09.15	09.16	0.0008

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TABLE 7
THE RESULTS OF THE ESTIMATED ΘW AS COMPARED TO THE PRACTICAL VALUES AT SAME DISTANCE (D=2M AND 3M).

Distance D (m)	Z (mm)	Practical view angle (θ_h)	Estimate view angle (θ_h)	Absolute percentage error
2	7	48.71	48.41	0.0062
	8	42.09	42.55	0.0110
	10	34.87	34.35	0.0149
	12	28.77	28.89	0.0041
	15	23.34	23.42	0.0035
	22	16.45	16.47	0.0015
3	7	48.20	48.41	0.0044
	8	40.88	42.55	0.0408
	10	35.04	34.35	0.0196
	12	28.10	28.89	0.0281
	15	23.19	23.42	0.0100
	22	16.2201	16.4694	0.0154

5 Conclusions

From the results can be concluding the following points:

1. The estimated mathematical model to calculate the viewing angles (θ_h and θ_w) for the digital cannon camera (power shot A800 and IXUS digital compact camera, 2011) can be status by:

$$\theta_h = 0.27 + \frac{195.38}{z}$$

$$\theta_w = 1.57 + \frac{327.88}{z}$$

2. This model gives high agreement between the practical value and stimated value.

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